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HEAT AND MASS TRANSFER BETWEEN GAS AND GRANULAR MATERIAL—PART II†

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Аннотация—В статье рассмотрены процессы переноса тепла и вещества в газовзвеси (от газа к частицам) и установлены основные факторы, влияющие на их интенсивность.

Дан анализ влияния на перенос: концентрации газовзвеси, гидродинамической и тепловой нестационарности, турбулизации набегающего потока, вращения частиц и их формы.

Обоснована система расчета процессов переноса в газовзвеси. Показано, что в газовзвеси мелких частиц процессы переноса протекают с весьма высокой интенсивностью, резко возрастающей при увеличении концентрации. Установлено, что торможения переноса за счет стесненности движения в газовзвеси мелких частиц при $Re < 10$ не может быть. Показаны возможности достижения высокой эффективности теплообменников и реакторов, использующих газовзвесь. Время нагрева газа (частиц) в условиях «гомогенной» газовзвеси может быть доведено до десятитысячных и стотысячных долей секунды.

NOMENCLATURE

- w_{fc}, cold filtration rate per unit total cross-section referred to 0°C (273°K);
- d, d_{eq}, diameter and equivalent diameter of a particle respectively;
- z, bed height, flow path;
- z/d, dimensionless bed height;
- f_p, bed porosity;
- γ_v, concentration of solid particles in gas suspension;
- v_B, terminal falling velocity;
- λ, thermal conductivity;
- a, thermal diffusivity;
- D, diffusivity;
- γ_s, γ_g, density of particles and gas;
- ξ, C_f, friction factor; drag coefficient;
- α, β, heat- and mass-transfer coefficients;

$$Re = \frac{w\,.\,d}{\nu}; \qquad Re_{eq} = \frac{w\,.\,d_{eq}}{\nu};$$

$$Re_z = \frac{w\,.\,z}{\nu}; \qquad Pr = \frac{\nu}{a};$$

$$Pr_{dif} = \frac{\nu}{D}; \qquad Pe = Re\,.\,Pr;$$

$$Nu = \frac{\alpha\,.\,d}{\lambda}; \qquad Nu_{dif} = \frac{\beta\,.\,d}{D};$$

$$\varphi = \frac{Nu}{Pe}; \qquad \varphi_{dif} = \frac{Nu_{dif}}{Pe_{dif}};$$

$$Bi = \frac{\alpha\,.\,d}{\lambda_s}$$

- Q, Q_a, amount of heat received (released) and amount of heat obtained at $\tau = \infty$, i.e. when heating particles up to equilibrium (final) temperature, respectively.

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1. HEAT AND MASS TRANSFER IN MOTION OF SOLID PARTICLES IN A GAS SUSPENSION

PART I [1] dealt with heat and mass transfer from solid particles in a fixed bed to a gas passing through it. Fundamental relations were established which allowed to conduct with sufficient accuracy appropriate calculation of interaction of a gas with solid particles and of heating (cooling) of a gas at Re from 30 to 400.

The design equations and the main dimensionless relations obtained for a fixed bed

$$Nu = 0{\cdot}24 \, . \, Re^{0{\cdot}82} \quad \text{and}$$

$$\varphi = 0{\cdot}32 \, . \, Re^{-0{\cdot}18} \qquad (1)$$

also remain valid for a moving packed bed if this bed of granular material moves as a compact mass without no mixing of particles in the volume in which heat- and mass-transfer processes proceed.

The mutual displacement of particles in a packed bed† may be ascertained using the

$$\xi_b = \frac{1800}{Re_{\text{fil}}} + \frac{48}{Re_{\text{fil}}^{0{\cdot}08}} \qquad (2)$$

which is valid and unique for fixed and moving packed beds. In case of "discontinuous" displacement of particles in a packed moving bed, equation (2) changes abruptly even with small increase of bed porosity.

The change in the friction factor dependence ξ_b on Re_{fil} is very characteristic when the type of bed [3] is changed. With a gas moving upwards through a packed bed of solid spherical particles, the friction factor varies with an increase in Re_{fil}, as is shown on curve II (Fig. 1). For non-spherical particles the curves $\xi_b = f(Re_{\text{fil}})$ are approximately equidistant and above curve II, depending on shape and roughness.

A small increase of gas velocity through a packed bed above its critical value disturbs the stability of the bed and produces a sharp change in the character of equation (2). The friction factor becomes approximately inversely proportional to Re_{fil} and strongly dependent on particle size; equation (2) is no longer single-valued, which was characteristic for a packed bed.

As has been already shown, in a packed bed the intensity of heat and mass transfer between a gas and a surface of solid particles is very high and for Re from 30 to 400 is given by equation (1).

† With separation of particles from each other and formation of a gap between them.

The slightest disturbance of a bed density leading to displacement "pouring" of particles sharply changes the hydrodynamic characteristics of a bed as well as strongly decreasing the transfer intensity, which cannot be described by equation (1).

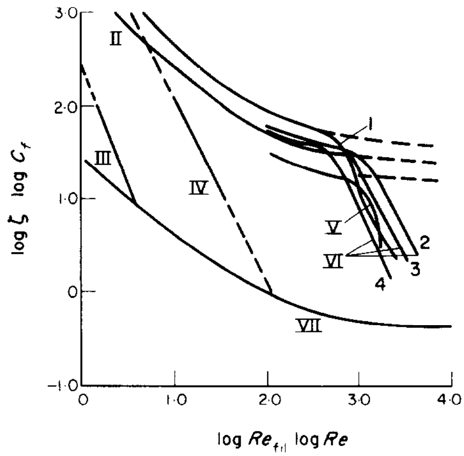

FIG. 1. Influence of Re upon friction factor for a packed bed:
I packed bed of particles of the Moscow Coal basin
II packed bed of spheres
III coal dust of the Moscow Coal basin (88–250μ)
IV powdered catalyst (250–640μ)
V steel spheres 3 mm dia.
VI chamotte and coal particles of different sizes, diameter of particles is measured in mm: 1·4–5; 2·6–7; 3·5–6; 4·4–5 (coal)
VII $C_f = f(Re)$ for sphere.

When the gas velocity is above its critical value the packed bed as the mode of interaction between a gas and granular material with all its inherent properties, in particular, with single-valuedness of equation (1) for different sizes of particles, vanishes converting into quite a new mode of interaction between granular material and a gas flow, which differs sharply from a packed bed.

This new mode of interaction is not stable since the change in the gas velocity leads to changes in "bed" porosity and the character of displacement of solid particles. At first, however, the motion of particles in a bed is, in general, hardly noticeable but, as the velocity increases, it becomes more intense and the bed becomes externally like a "boiling" liquid, hence it is called a "fluidized" bed.

For a long period of time it has been assumed that a "fluidized bed" is characterized by the highest intensity of heat and mass transfer between a gas and solid particles.

Meanwhile in reality, while passing from a packed bed to a "fluidized" one, the intensity of heat and mass transfer greatly decreases (usually several times). This is clearly seen in Fig. 2 where are presented the curves of oxygen flow rate in a bed of burning carbon particles $\sim$ 3·2 mm in size [2]. Curve I is for a packed bed with porosity $f_p = 0{\cdot}5$ and velocity $w_{fl1} \sim 0{\cdot}25$ m/s. Curves II, III and IV are for a fluidized bed with porosities and velocities, respectively: 0·56 and 0·42 m/s; 0·62 and 0·52 m/s; 0·68 and 0·62 m/s [2]. In this figure are also presented the points for a packed bed with porosity 0·5 and velocity 0·61 m/s.

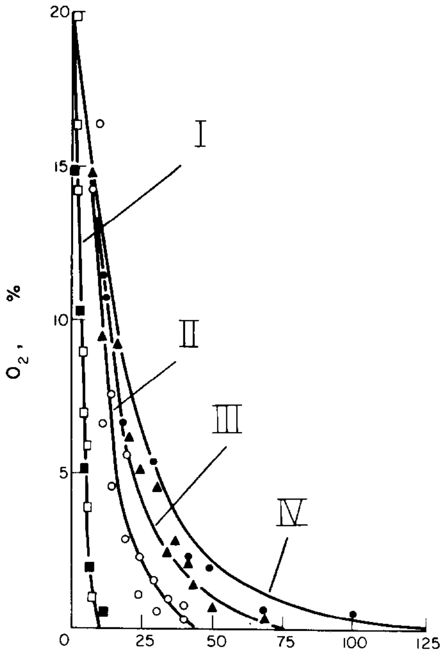

FIG. 2. Dynamics of oxygen flow rate with respect to height of fluidized and packed beds at different conditions:
fluidized bed: d = 2·6–3·7 mm
○ w_{fc} – 0·42 m/s; ▲ w_{fc} – 0·52 m/s;
● w_{fc} – 0·62 m/s
packed bed: d = 2·6–3·7 mm
□ w_{fc} – 0·61 m/s;
d = 4–5 mm
■ w_{fc} – 0·25 m/s.

In Fig. 2 it is seen how sharply the rate of combustion decelerates which is an indication of the mass-transfer rate under these conditions.

The greatest jump with a decrease in mass transfer is observed when changing a bed structure from curve I to curve II despite the fact that the value of the reaction surface of carbon F_{react} in a bed decreases only by 12 per cent and, as is seen from Fig. 3, the distance between the adjacent particles practically does not vary.

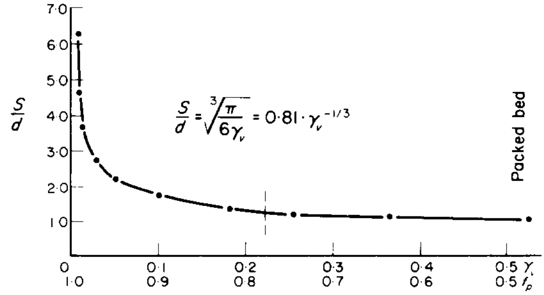

FIG. 3. Influence of volumetric concentration of solid particles in a gas suspension upon distance between particles.

In the packed bed of burning carbon particles usually the size of the oxygen zone, i.e. the bed height over which oxygen concentration decreases up to 5–10 per cent of its initial value, does not exceed two rows of particles, $z/d < 2$. Under the same conditions in a uniformly fluidized bed the size of the oxygen zone increases 6–10 times. This means that firstly equation (1) is quite inapplicable to determine Nu and φ in a fluidized bed and secondly considerably smaller values of Nu and φ satisfy the same values of Re_{fl1} for a fluidized bed.

The increase in the gas velocity (upwards) in a fluidized bed† leads to increase in its porosity. For a certain critical velocity equal to that of free fall of solid particles the fluidized bed converts into a gas suspension when relative motion

† For simplicity we consider disperse material of uniform particle size.

of particles ceases.† With upward flow of gas, when the gas velocity is above that of free-fall of particles in a gas suspension, the gas and the solid particles move in one direction with the relative velocity close or equal to that of free-fall of these particles under steady-state conditions.

Gas suspension may be non-uniform when particles are poorly mixed with the gas and uniform "homogeneous" when solid particles are statistically uniformly distributed over the whole gas volume and the "concentration" of solid particles γ_v in m^3 per m^3 of gas suspension is statistically invariable in the whole "active space" where heat- and mass-transfer processes are taking place. In this paper only homogeneous gas suspension of uniform particle size is considered otherwise specified.

2. GAS SUSPENSION

The arrangement of flow of heat transfer media in a gas suspension may be countercurrent, cocurrent and crosscurrent (transverse).

The countercurrent arrangement of gas suspension is best obtained in the form of a "falling bed" when the gas flows upwards, with a velocity smaller than the velocity of free-fall of the particles v_B, through a "rain" of solid particles falling uniformly over the cross-section of a chamber.

The cocurrent flow arrangement of gas suspension is possible with motion of gas and particles downwards and upwards and under certain conditions in eddy, cyclone flows, etc.

The transverse flow arrangement of gas suspension is, in particular, possible with transverse gas motion through the falling bed of solid particles or in rotary flow.

The volumetric concentration of a solid phase in gas suspension may vary over wide ranges from 10^{-6}–10^{-4} to more than 0·3 m^3 of solid particles per m^3 of gas suspension.

In industry and power generation the use of heat and mass transfer in gas suspension (particle ⇄ gas) is increasing from year to year since this mode of heating is very effective [4]. The heating of air in gas suspension is carried out up to high temperatures, and this is very important for new methods of production of electrical energy and for a number of new technological processes.

Despite great practical necessity and a great number of investigations including very thorough and valuable ones [3], up to now there is no reliable theoretical basis for solving the problem on heat and mass transfer in a gas suspension.

In some cases the problem on heat and mass transfer both for an individual particle and for a gas suspension may be exactly formulated theoretically by writing down the corresponding system of the equations. However, very often this problem cannot be theoretically solved due to its complexity, and therefore to obtain a method of calculation experimental data and semi-empirical relations should be mainly used.

Unfortunately the complexity of a heat and mass transfer process in a gas suspension also restricts the use of experimental data since there is no reliable model of a mechanism for the transfer phenomena, and the experimental data are contradictory and not always reliable. Up to now there are no generally accepted dimensionless equations governing the process intensity and, consequently, there are no methods for calculating heat exchangers and reactors.

This circumstance essentially keeps back the development of the practical application of the method of calculation, experimental data and laboratory engineering to study exo- and endothermal processes and reactions.

Very different results obtained in a number of experimental investigations on intensity of transfer processes in gas suspension give rise to doubt and uncertainty among engineers and design workers. Very often these differences exceed hundreds of per cent and attain even a whole order of magnitude.

The problem of the effect of concentration of gas suspension upon transfer intensity is quite obscure. Some specialists [3] consider that, for

† Displacement due to turbulent pulsation is not taken into account.

all practical purposes, concentration of the solid phase starts considerably decreasing the transfer intensity already from $3 \cdot 10^{-4}$ m^3 per m^3 of gas suspension. The author of the present paper does not share this point of view but considers that concentration can influence a process only starting from values which are two orders of magnitude higher.

Moreover, even the main problem, whether there are any differences in heat transfer between the gas and individual stationary spherical particles and those of another shape and of particles in "flight" in a gas suspension system is not uniquely solved up to now.

At present there are a great number of publications available on heat and mass transfer from a gas in flow to stationary spherical particles and other bodies [5]. Works on heat and mass transfer from particles to a gas in a gas suspension are also known. Before considering the experimental results let us study briefly a physical pattern of interaction between a gas flow and particles and try to find the similarity and the difference between the interaction of a gas and "free-falling" particles and between a flowing gas and stationary particles.

The character of gas motion near the surface of stationary spheres and cylinders has been studied many times. For a small *Re* (~ 1) the character of gas flow past a sphere is laminar and non-separated but already at $Re \approx 200$ at the rear part of a sphere as well as of a cylinder it is possible to observe the separation of the boundary layer and the formation of a "vortex" path with reverse gas flow past the rear part of the sphere (cylinder) [6].

With an increase in *Re* the eddy formation at the rear part of a body flow increases, and the flow past the frontal part of a sphere (cylinder) remains by its nature invariable, and the boundary layer at low turbulence of an incoming flow remains laminar up to $Re \approx 10^5$, at which there occurs turbulization of the boundary layer. Such is, in general, the scheme of a physical pattern of interaction between individual stationary bodies and gas flow.

What is the difference between this pattern and that of interaction between a gas flow and particles in gas suspension? First of all, the mutual effect of adjacent particles upon a gas flow past them is, in principle, possible and even inevitable. Obviously this effect is the more pronounced, the higher the volumetric concentration of gas suspension γ_v. The presence of the adjacent particles may also influence the interaction between a gas and a particle due to particle collisions.

A free particle in a gas suspension may move in flow in different directions, and it is especially important that almost always, apart from translational motion it may rotate. The rotation may be, in addition, non-uniform with possible changes in the orientation of the rotation axis. All this will inevitably promote the onset of pulsations and the artificial increase in "incoming" flow turbulence on separate particles as well as their unsteady interaction.

Thus, a particle in a gas suspension will interact with a gas flow differently from a stationary particle and the interaction depends on many reasons which are very peculiar and complex. At present this problem cannot be solved theoretically, and the only way of its approximate solution consists of estimating and taking into account the effect of separate factors (rotation, concentration, unsteadiness, flow turbulence, etc.) and their totality under real conditions of heat and mass transfer from a gas to a particle in gas suspension.

In order to obtain a more exact estimate, consider in more detail a process of interaction between a gas and a stationary particle and elucidate the effect of separate factors peculiar to the flow of a gas suspension upon this process.

3. INTERACTION OF GAS FLOW WITH STATIONARY SPHERICAL PARTICLE AND PARTICLE OF DIFFERENT CONFIGURATION

A heated metallic sphere is introduced into a cold gas flow. With no heat sources the heat conduction equation for isotropic materials with symmetric cooling of a sphere may be written as follows

$$\frac{\partial T}{\partial \tau} = a \,.\, \nabla^2 T. \qquad (3)$$

Solving this equation under the appropriate boundary and initial conditions it is possible to find a temperature distribution at any point of the sphere for the time τ. At high values of thermal conductivity of sphere material or more exactly at small values of the Biot number ($\overline{Bi} < 0{\cdot}1$†) the equation determining cooling (heating) time of a sphere up to a definite state which can be characterized by the relation of the amount of heat released by a sphere Q to that which can be transferred Q_a at $\tau = \infty$ is determined by the following simple equation at $T_{gas} = \text{const.}$ [2]

$$\tau(Q/Q_a) \approx 1380 \,.\, C_s \,.\, \gamma_s \, \frac{d}{\alpha} \,.\, \log\left(1 - \frac{Q}{Q_a}\right). \qquad (4)$$

From the experimental data on sphere cooling it is possible, using equation (4), to determine mean values of the heat transfer coefficient α for any time and, vice versa knowing α, it is not difficult to calculate the curves for cooling a sphere.

For $Bi < 0{\cdot}1$ equation (4) is also applicable for calculating the time of cooling of bodies (particles) of another configuration using the diameter (d_{eq}) of particles.

Figure 4 presents experimental data on cooling of metallic spheres and a cube in a gas flow [7]. The curves are calculated for a value of α invariable in time. The accuracy of the experiments is quite satisfactory, which is clearly demonstrated by the curves plotted at α differing by ± 5 per cent with reproducibility of experimental points of ± 3 per cent.

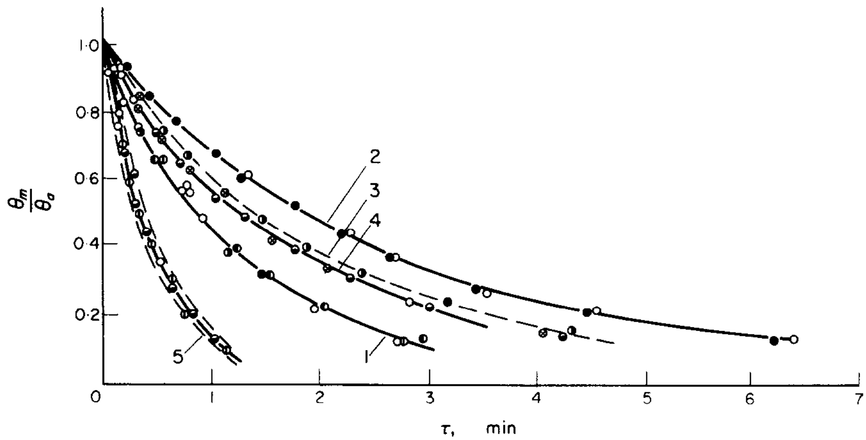

FIG. 4. Change in temperature of spheres and cubes in air flow

1. cube: $a = 20$ mm $Re = 32750$ $\alpha = 158$ kcal/m²h°C
2. sphere: $d = 19{\cdot}8$ mm $Re = 4185$ $\alpha = 72$ kcal/m²h°C
3. sphere: $d = 29{\cdot}4$ mm $Re = 54100$ $\alpha = 140$ kcal/m²h°C
4. sphere: $d = 25{\cdot}4$ mm $Re = 34100$ $\alpha = 132$ kcal/m²h°C
5. sphere: $d = 12{\cdot}5$ mm $Re = 23240$ $\alpha = 185$ kcal/m²h°C

Dashed curves 5 are plotted at α differing from $\alpha = 185$ kcal/m²h °C by 5 per cent.

The above curves are characterized by the constancy of α in a heat transfer process, although theoretically at the initial time when a thermal boundary layer is not yet formed and a temperature gradient is high, α should have considerably higher values.

It is obvious that in these experiments the value of the initial unsteady heat transfer period is quantitatively not high, therefore it does not influence the character of the curves of cooling relatively large spheres and cubes by air (Fig. 4).

† Under these conditions the sphere temperature is uniform at its centre and near its surface; the temperature gradient is small.

A change in the hydrodynamic conditions leads to a change in the intensity of heat transfer from a surface to a gas. In a fixed infinite gas medium the intensity of cooling a sphere† is determined by

$$Nu \approx \frac{\alpha . d}{\lambda} \approx 2. \tag{5}$$

This equation is also valid for a gas flow at small Re not exceeding 1–2. An increase in Re leads to an increase in Nu. The empirical equation for Re from 1 to 150–200, i.e. for non-separated flow past a sphere, which satisfactorily describes the experimental data, is of the form

$$Nu \approx 2 + 0{\cdot}4 . Re^{0{\cdot}5}. \tag{6}$$

With a further increase in Re above 200 for separated flow past a sphere when there occurs formation of a laminar boundary layer and up to $Re = 4\text{–}7 . 10^3$ the dimensionless equation for a fixed sphere is simplified to

$$Nu \approx 0{\cdot}55 . Re^{0{\cdot}5}. \tag{7}$$

The experimental data obtained without intensifying the incoming flow turbulization are quite satisfactorily described by this equation [8].

It has been already mentioned that the theoretical solution of such a problem on heat transfer between a gas and sphere is extremely difficult; however, for another "external" problem on heat transfer from a plate (slab) it was obtained exactly for the conditions of the formation of the laminar Pohlhausen boundary layer [9]. The equation obtained by Pohlhausen may be written for a mean Nu number over a plate as follows

$$Nu_z = 0{\cdot}668 . Re_z^{0{\cdot}5} . Pr^{0{\cdot}33} \approx 0{\cdot}61 . Re_z^{0{\cdot}5}. \tag{8}$$

In Fig. 5 are presented plots of Nu vs. Re for external flow past bodies of different configuration: plate, sphere, cylinder and cube. For all these problems the experimental data obtained with a change in Re within 100 and $4\text{–}7 . 10^3$ lie well on the curves where Nu is proportional to $Re^{0{\cdot}5}$, that according to the Pohlhausen solution corresponds to the formation of a laminar boundary layer at a surface.

† Without regard for heat removal due to radiation and natural convection.

The second important peculiarity which is clearly seen from Fig. 5 is the fact that heat transfer between a gas and cylinder and that between a gas and sphere differ only slightly according to the curve $Nu = f(Re)$. For heat transfer between a gas and bodies of different configuration such as a plate, cube, etc, the value of the heat transfer coefficient is not comparatively great.

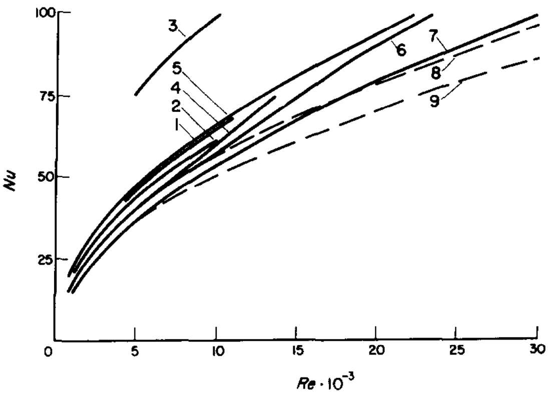

FIG. 5. Heat transfer between gas and bodies of different configuration.
1—disc, 2—plate ($Pr = 0{\cdot}7$), 3—sphere in strongly turbulized incoming flow, 4—cubes, 5—plate ($Pr = 1{\cdot}0$) 6—sphere, 7—cylinder, 8—$Nu = 0{\cdot}56 . Re^{0{\cdot}5}$, 9—$Nu = 0{\cdot}5 . Re^{0{\cdot}5}$.

For the region with Nu proportional to $Re^{0{\cdot}5}$, i.e. for a laminar boundary layer and the same degree of turbulence of the incoming flow the values of Nu at the same Re for bodies of different configuration differ mainly within ± 10 per cent from those of Nu for a sphere.

The following fact is undoubtedly of interest that heat transfer with impact of a gas jet on a disc surface (transverse flow past a disc) corresponds to that between a gas and a surface in a laminar boundary layer when $Nu \approx A . Re^{0{\cdot}5}$ [10].

When treating Day's data [11] on combustion of a carbon hole the author [12] has established that, even with high-rate injection of air into the hole through a nozzle, a laminar boundary layer is formed near the hole surface in reverse gas flow, and the intensity of oxygen transfer to a carbon surface is well described by Pohlhausen's equation (8).

of flow past a sphere and cylinder despite the fact that the reverse "incoming" flow of a gas has pronounced eddy nature. For this very reason heat transfer between a sphere and cylinder at $Re \sim 4\text{–}7 . 10^3$ is determined by the

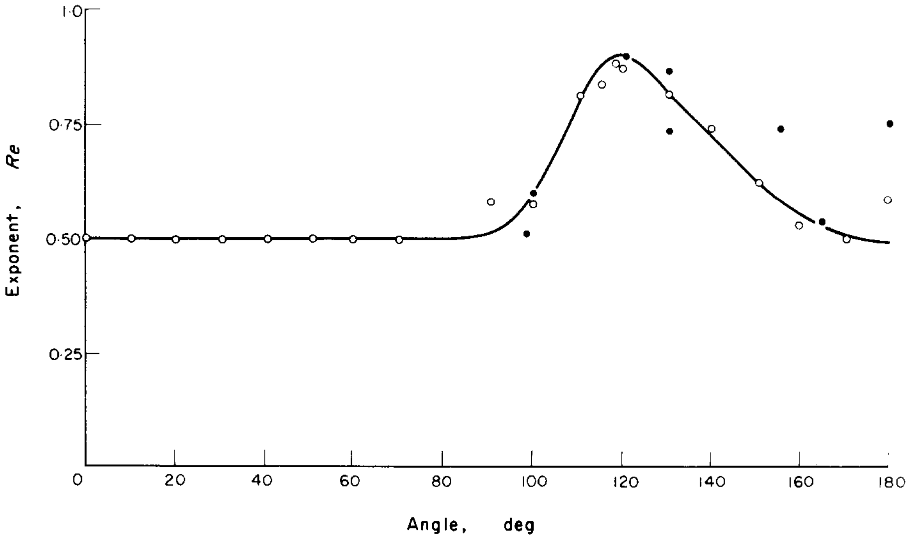

FIG. 6. Dependence of the exponent of the Reynolds number upon polar angle at sphere (according to T. R. Galloway and B. H. Sage).

The data presented here and numerous results of other investigations on heat and mass transfer convincingly confirm that, when forming a laminar boundary layer in flow past bodies of any configuration, in the dimensionless transfer equation: firstly, a change in Nu is proportional to $Re^{0.5}$ and secondly, the configuration of bodies which interact with a gas flow influences comparatively slightly the absolute values of the Nu number.

This fact is convincingly confirmed by the experimental data of Work [13] on heat transfer between a "sharp" gas jet and a hole forming in a liquid, into which this jet is injected with high velocity. In this case the high jet velocity and pulsation of the walls of the hole, from which there occurs mass transfer, the boundary layer is laminar and transfer intensity (Nu) is proportional to $Re^{0.5}$.

The formation of a laminar boundary layer at the rear is no less peculiar for the conditions equation of type (7). The aforesaid is well illustrated by the curve in Fig. 6 [14] with a change in the exponent of the Reynolds number over a sphere surface in a gas flow starting from a front "stagnation" point (0°) up to a "stagnation" point of the rear† part of a sphere (180°).

As is seen from the curve and the points in Fig. 6, for the whole frontal part of a sphere up to the place of boundary layer separation the Reynolds number has the exponent of 0·5 over the whole range of Re up to critical values. This very exponent is repeated at the "frontal" part of reverse flow at the rear part of a sphere.

What happens to the heat and mass transfer with turbilization of a boundary layer when in addition to molecular transfer in a pseudo-laminar boundary sublayer there appears molar transfer by turbulent pulsations?

In this case there is no theoretical solution

† The rear point (180°) of the main flow.

even for the simplest problem, not speaking of such a complex problem as heat and mass transfer from a gas to a sphere or cylinder.

However, there are semi-empirical equations for heat transfer between a gas and a plate in a turbulent boundary layer. As a rule these equations take into account the fact that at the initial section of the plate along the length z under normal conditions the boundary layer exists in which transfer intensity is determined by equation (8).

For a turbulent boundary layer, according to Johnson [6], the equation for heat transfer between a gas and a plate is

$$Nu_z\,0{\cdot}037 \,.\, Re_z^{0{\cdot}8} \,.\, Pr^{1/3}; \quad \varphi = 0{\cdot}047 \,.\, Re_z^{-0{\cdot}2}. \tag{9}$$

On the basis of the hydrodynamics theory of heat transfer the author [15] arrives at the equation

$$\frac{Nu}{Pe} = \frac{\xi}{8} \cdot \frac{2Q}{1 - (w_1/w)^2} = \frac{\xi}{4} \cdot \frac{1}{1 + (w_1/w)\,[2Pr - (w_1/w)]} \tag{10}$$

which for local values of Nu in case of flow past a plate for the case of a turbulent boundary layer is written as

$$Nu_z = \frac{0{\cdot}058 \,.\, Re_z^{0{\cdot}8}}{1 + 4{\cdot}02\, Re_z^{-0.1} - 4{\cdot}05 \,.\, Re_z^{-0.2}} \,. \tag{10*}$$

For mean values of Nu_z and φ equation (10*) is approximated by

$$Nu_z \approx 0{\cdot}028 \,.\, Re_z^{0{\cdot}82}; \quad \varphi \approx 0{\cdot}039 \,.\, Re_z^{-0{\cdot}18}. \tag{11}$$

When laminar and turbulent layers are present, the dimensionless equation for heat transfer is written† as follows

$$Nu_z \approx 0{\cdot}61 \,.\, Re_z^{*0{\cdot}5} + 0{\cdot}028 \,.\, Re_z^{0{\cdot}82} - 0{\cdot}028 \,.\, Re_z^{*0{\cdot}82} \tag{12}$$

† Since $\bar{\alpha} \,.\, z = \bar{\alpha}_1 z^* + \bar{\alpha}_2 (z - z^*)$, then $\overline{Nu}_{z\,\mathrm{tot}} = \overline{Nu}_{z\,\mathrm{tur}} + \overline{Nu}_{z^*\mathrm{transient}} - \overline{Nu}_{z\,\mathrm{tur}}$: and consequently $\overline{Nu}_{z\,\mathrm{tot}} = \overline{Nu}_z = 0{\cdot}028\ (Re_z^{0{\cdot}82} - Re_{z*}^{0{\cdot}82}) + 0{\cdot}061\ Re_z^{0{\cdot}5}$ or according to Johnson: $\overline{Nu}_{z\mathrm{tot}} \approx \overline{Nu}_z \approx 0{\cdot}037\ (Re_z^{0{\cdot}8} - Re_{z*}^{0{\cdot}8}) + 0{\cdot}061 \,.\, Re_z^{0{\cdot}5}$.

where Re_z^* is the Reynolds number satisfying a laminar boundary layer for which z^* is the characteristic length from which a turbulent boundary layer starts. The Re_z^* number for a heated plate is ~ 100000, and in this case equation (12) is simplified to

$$Nu_z = 0{\cdot}028 \,.\, Re_z^{0{\cdot}82} + 193–348 = 0{\cdot}028 \,.\, Re_z^{0{\cdot}82} - 155. \tag{13}$$

Figure 7 gives a comparison of equation (13) and experimental data of heat transfer between flow and a slab (plate). As is seen from Fig. 7, the experimental points are satisfactorily correlated by equation (13).

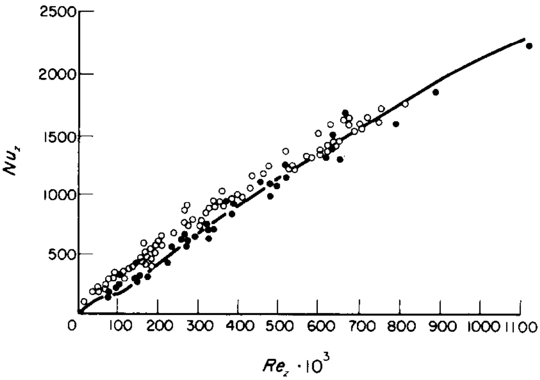

FIG. 7. Heat transfer between gas and slab [Predicted curve is plotted by equation (13).]
⊙ —Frank's experimental points
○ —Jurges' experimental points
● —Elias' experimental points.

Meanwhile, the analysis of equation (12) shows that under favourable conditions in a boundary layer the turbulent boundary layer at a plate may appear at considerably smaller values of Re_z than is usually accepted.

Assuming that both the character, the regularity and the quantitative characteristics of heat transfer between a gas and a cylinder and plate are practically identical, according to Pohlhausen's equation, we have for non-separated flow past a cylinder

$$Nu = \frac{0{\cdot}61}{\sqrt{(\pi/2)}} \,.\, Re^{0{\cdot}5} = 0{\cdot}5 \,.\, Re^{0{\cdot}5}. \tag{14}$$

Taking into account the fact that with separation of a boundary layer the characteristic length is $\sim 0{\cdot}72d$; the heat-transfer equation for a laminar boundary layer is written as follows:

$$Nu = 0{\cdot}36 \,.\, Re^{0{\cdot}5} + a_1 \,.\, 0{\cdot}36 \,.\, Re^{0{\cdot}5}_{\text{back}}. \quad (15)$$

The second term in this equation depends on the size of the active surface of the rear part of a cylinder and on the velocity of the reverse flow.

Since the velocity of the reverse flow is proportional to that of the forward incoming gas flow then $Re_{\text{back}} = K \,.\, Re$ and, consequently the equation may be written as:

$$Nu = 0{\cdot}36 \,.\, Re^{0{\cdot}5} + a_2 \,.\, Re^{0{\cdot}5} \approx 0{\cdot}5 \,.\, Re^{0{\cdot}5}. \quad (16)$$

On the basis of numerous investigations [6] $a_2 = 0{\cdot}14$. This value of a_2 at a constant magnitude of the active surface corresponds to the velocity of the reverse flow, which is approximately 6 times smaller than that of the incoming flow.

All known investigations uniquely show that at the frontal part of a cylinder and sphere the laminar boundary layer (up to the separation point) is very stable and remains so up to large *Re* under ordinary conditions. However, the nature of the laminar boundary layer does not exclude the possibility of influencing its thickness and, consequently, the transfer process intensity by other ways, apart from increasing *Re*. One of these ways is an increase in the turbulence of the incoming flow [14], that intensifies transfer but does not change the exponent of the Reynolds number over a wide range of *Re* in the dimensionless heat-transfer equation.

The laminar boundary layer is less stable when appearing at the rear part of a cylinder and sphere and becomes turbulent already at $Re_z = 5 \,.\, 10^3\text{–}10^4$. Undoubtedly the eddy character of the reverse flow promotes this.

The whole pattern of the interaction of flow with a cylinder† sharply changes at the rear part when increasing *Re* above $4 \,.\, 10^3\text{–}10^4$ when a boundary layer turbulizes in accordance with the three-term equation (12). The equation for heat transfer between a gas and a cylinder in this case will be rather complex because the rear surface will be covered with a laminar boundary layer. Since the motion at the rear part is due to the eddy formation and periodic separation of eddies, then it may be considered that the turbulent boundary layer at the rear part formed near the point of separation of a reverse boundary layer should immediately spread almost over the whole rear surface of a cylinder.

In this case equation (12) assumes a comparatively simple form

$$Nu \approx 0{\cdot}36 \,.\, Re^{0{\cdot}5} + b \,.\, 0{\cdot}028 \,.\, Re^{0{\cdot}82}. \quad (17)$$

where the coefficient b accounts both for a part of the surface of a cylinder occupied by the turbulent boundary layer and for a decrease in the velocity of the reverse gas flow.

Thus, without essential inaccuracies and assumptions the dimensionless heat transfer equation for a sphere, cylinder, cube and other bodies of regular and irregular shape may be written as:

$$Nu \approx A \,.\, Re^{0{\cdot}5} + B \,.\, Re^{0{\cdot}82}. \quad (18)$$

The first term of the equation represents heat transfer† in the laminar and the second term in the turbulent sections of the boundary layer as a result of the interaction between flow and a stationary solid.

The analysis of the general equation obtained in the light of the above mechanism of heat and mass transfer as well as of interaction between flow and a particle allows the main trends and the ways of effective intensification of heat transfer to be established.

Before dwelling upon the analysis of equation (18) in detail, let the region of *Re* values be determined which are of practical and theoretical interest for a gas suspension.

As is known, the intensity of heat and mass transfer in a gas suspension is determined not

† With a sphere and other bodies in case of flow separation.

† Mass transfer too.

only by α and β and, consequently by $Nu(\varphi)$ and Nu_{dif} (φ_{dif}) but also by the magnitude of the surface area per unit volume, i.e. by the size of particles d_{eq} and their concentration γ_v. The finer the particles in a gas suspension and the greater γ_v, the greater the surface area per unit volume S_p which for spherical particles is

$$S_p = \frac{6 \cdot \alpha_v}{d}. \quad (19)$$

While gas suspension concentration γ_v is practically limited by a maximum value of the order of 0·2–0·3 the size of particles may vary within very great ranges from 50–100 mm down to 0·01–0·02 mm.

It is not difficult to see from equation (19) that practical interest gas suspensions with particles less than 4–6 mm in size for creating high transfer intensities per unit volume exists only for when the surface area S_p is sufficiently large. These conditions apply to laminar flow ($Re \leqslant 10^4$). A still more narrow range of the Reynolds number (up to 500–1200) is of the most practical interest for industrial and laboratory conditions.

From the aforesaid it is clear that the motion of particles in these cases differs from the so-called supercritical regime of flow when in "normal" conditions the boundary layer turbulizes at the frontal part of a sphere, cylinder and other bodies. Moreover, in the Re region of interest under "normal" conditions at the rear part of a sphere and cylinder there exists mainly a laminar boundary layer.

Before studying individual ways of transfer intensification and characteristic peculiarities of gas suspension, consider briefly the formal characteristic of equation (18) at relatively small Re.

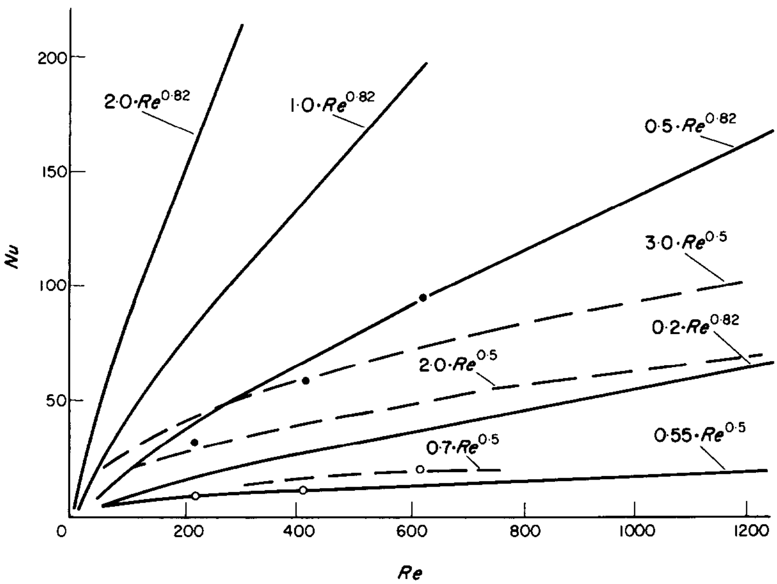

FIG. 8. Intensification of heat and mass transfer by artificial turbulization and "compression" of laminar boundary layer.
● –Slobodkin's experimental points, unsteady drying
○ –Slobodkin's experimental points, steady drying.

Figure 8 shows the character of intensification of heat and mass transfer and its effect upon the coefficients A and B in equation (18). The lower curve in Fig. 8 satisfies the "normal" equation (8) for heat transfer between gas flow and a stationary spherical particle, and the curve

$0.2 . Re^{0.82}$ corresponds to the equation for heat transfer between gas and particles in a gas suspension [16]. The remaining curves are plotted at $A = 2$ and 3 and $B = 0.5$, 1·0 and 2·0. A change in the coefficient A satisfies the "compression" of a laminar boundary layer, i.e. a decrease in the thickness of a boundary layer not due to an increase in Re. The increase in the coefficient B is the same for a turbulent boundary layer, which corresponds to an earlier transition to a turbulent regime of flow.

Part III will deal with the nature of intensification of heat and mass transfer in a gas suspension.

First of all, one of the main problems, i.e. the effect of concentration γ_v upon transfer processes, will be considered.

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TRANSPORT DE CHALEUR ET DE MASSE ENTRE UN GAZ ET UN MATERIAU GRANULAIRE

Résumé—Les processus de transport de chaleur et de masse dans une suspension gazeuse (depuis un gaz jusqu'aux particules) sont considérés, et les facteurs principaux influençant leur intensité sont établis dans l'article.

L'influence de la concentration de la suspension gazeuse, de l'instationnarité hydrodynamique et thermique, de la mise en turbulence de l'écoulement d'entrée, de la rotation des particules et de leur forme sur le transport est analysé.

Le système de calcul des processus de transport dans une suspension gazeuse est établi. On a montré que les processus de transport dans une suspension gazeuse de particules fines se produisent avec une intensité très élevée, augmentant fortement avec la concentration. On a établi qu'il ne peut y avoir aucune décélération du transport due au mouvement de pression dans une suspension gazeuse de particules fines à $Re < 10$. Les moyens d'obtenir un rendement élevé d'échangeurs de chaleur et de réacteurs employant des suspensions gazeuses sont indiqués. Le temps de chauffage d'un gaz (particules) dans une suspension gazeuse "homogène" peut aller jusqu'à dix et cent millisecondes.

WÄRME- UND STOFFAUSTAUSCH ZWISCHEN GAS UND FEINKÖRNIGEM MATERIAL

Zusammenfassung—Wärme- und Stoffaustauschvorgänge in einer Gas-Suspension (vom Gas an die Teilchen) werden betrachtet und die hauptsächlichen Faktoren, die die Intensität dieser Vorgänge beeinflussen, angegeben.

Der Einfluss der Konzentration der Gas-Suspension, der hydrodynamischen und thermischen Unregelmässigkeit, sowie des Turbulenzgrades der ankommenden Strömung, der Rotation der Teilchen und ihrer Form auf den Wärme- und Stoffaustausch wird analysiert.

Das System der vermuteten Übertragungsvorgänge in Gas-Suspensionen wird dargestellt. Es konnte gezeigt werden, dass in Gas-Suspensionen mit kleinen Partikeln die Übertragungsvorgänge mit einer sehr hohen Intensität ablaufen, die sich mit wachsender Konzentration rasch erhöht. Ausserdem wurde festgestellt, dass bei einer gedrosselten Bewegung in Gas-Suspensionen mit kleinen Partikeln bei *Re* 10

mit Gas-Suspensionen einen hohen Wirkungsgrad zu erzielen, werden aufgezeigt. Die Zeit, um ein Gas (oder die Partikel) in "homogenen-" Gas-Suspensionen aufzuheizen, kann bis herunter zu Zehntausendstel oder Hunderttausendstel Sekunden betragen.

Abstract—Heat and mass transfer processes in gas suspension (from a gas to particles) are considered, and the main factors influencing their intensity are established in the paper.

The influence of gas suspension concentration, hydrodynamic and thermal unsteadiness, turbulization of incoming flow, rotation of particles and their shape upon transfer is analysed.

The system of calculating transfer processes in gas suspension is substantiated. It has been shown that in gas suspension of fine particles transfer processes occur with very high intensity, sharply increasing with concentration. It has been established that there may take place no transfer deceleration due to squeezed motion in gas suspension of fine particles at $Re < 10$. The ways of achieving high efficiency of heat exchangers and reactors using gas suspension are shown. The time of heating a gas (particles) in "homogenous" gas suspension may be up to ten thousandth and hundred thousandth fractions of a second.